# Spontaneous generation of spin-orbit coupling in magnetic dipolar Fermi gases

T. Sogo[1], M. Urban[1], P. Schuck[1,2], T. Miyakawa[3]
[1] *Institut de Physique Nucléaire, CNRS-IN2P3 and Université Paris-Sud, 91406 Orsay Cedex, France*
[2] *Laboratoire de Physique et Modélisation des Milieux Condensés, CNRS and Université Joseph Fourier, 25 Avenue des Martyrs, Boîte Postale 166, F-38042 Grenoble Cedex 9, France and*
[3] *Faculty of Education, Aichi University of Education, Hirosawa 1, Igaya-cho, Kariya 448-8542, Japan*

The stability of an unpolarized two-component dipolar Fermi gas is studied within mean-field theory. Besides the known instability towards spontaneous magnetization with Fermi sphere deformation, another instability towards spontaneous formation of a spin-orbit coupled phase with a Rashba-like spin texture is found. A phase diagram is presented and consequences are briefly discussed.

Recently, Bose-Einstein condensation of Dysprosium 164 has been achieved [1]. $^{164}$Dy is an atom with magnetic dipole moment $10\mu_B$ ($\mu_B$ being the Bohr magneton). Also a dipolar Fermi gas was recently produced in the form of a gas of fermionic molecules with large electric dipole moment [2]. This gas was, however, not degenerate. But since there exists a fermionic Dy isotope, $^{163}$Dy, we can expect in the near future the realization of a degenerate Fermi gas of atoms with large magnetic moment.

Dipolar atoms interact via the dipole-dipole interaction (DDI), which is anisotropic and long-ranged. In Fermi gases, this interaction has the effect to deform the Fermi surface. This phenomenon has been studied in Refs. [3–5]. In nuclear physics, the so-called tensor force has a similar structure as the DDI [6]. The effect of the tensor force in nuclear matter has been investigated in the framework of Landau Fermi liquid theory, e.g., with regard to the spin susceptibility [7, 8].

Since their recent realization [9], cold atomic systems with artificial spin-orbit coupling (SOC), in two as well as three dimensions, have received tremendous attention (see, e.g., [10–12]). In this letter, we show that in three dimensional unpolarized dipolar Fermi gases, the DDI can give rise to an instability towards spontaneous formation of a phase with SOC. This could be an alternative way, maybe simpler than the artificial SOC, to produce SOC in ultracold Fermi gases, and opens wide perspectives which have been intensely discussed in the very recent literature. For instance, SOC may have important consequences for pairing [11, 12].

The possibility of a spontaneous generation of a SOC phase will be studied within the mean-field and random-phase approximation (RPA) approach. We will first formulate the theory for a magnetic dipolar Fermi gas with arbitrary spin $s$ and then specialize to $s = 1/2$.

The magnetic DDI between two atoms with dipole moments $\mathbf{d}_1$ and $\mathbf{d}_2$ is given by

$$V_{dd}(\mathbf{r}) = -\frac{3}{r^3}\Big(\frac{(\mathbf{r}\cdot\mathbf{d}_1)(\mathbf{r}\cdot\mathbf{d}_2)}{r^2} - \frac{\mathbf{d}_1\cdot\mathbf{d}_2}{3}\Big)\,, \qquad (1)$$

where $\mathbf{r}$ is the distance between the atoms.

Let us consider a uniform gas of fermionic atoms having a magnetic dipole moment $\mathbf{d} = d_0\mathbf{s}$, where $\mathbf{s}$ is the spin operator and $d_0$ characterizes the magnitude of the dipole moment. The Hamiltonian is given by (we use units with $\hbar = 1$)

$$\begin{aligned} H = &\sum_\alpha \int \frac{d^3k}{(2\pi)^3}\frac{k^2}{2m} c^\dagger_{\mathbf{k},\alpha} c_{\mathbf{k},\alpha} \\ &+ \frac{1}{2}\sum_{\alpha\beta\alpha'\beta'}\int \frac{d^3k}{(2\pi)^3}\frac{d^3k'}{(2\pi)^3}\frac{d^3q}{(2\pi)^3} V_{\alpha\beta,\alpha'\beta'}(\mathbf{q}) \\ &\qquad\times c^\dagger_{\mathbf{k}+\mathbf{q},\alpha} c^\dagger_{\mathbf{k}'-\mathbf{q},\beta} c_{\mathbf{k}',\beta'} c_{\mathbf{k},\alpha'}\,, \end{aligned} \qquad (2)$$

The interaction includes the contact interaction as well as the dipole-dipole interaction: $V_{\alpha\beta,\alpha'\beta'}(\mathbf{q}) = V^c_{\alpha\beta,\alpha'\beta'} + V^{dd}_{\alpha\beta,\alpha'\beta'}(\mathbf{q})$.

The contact interaction is written as $V^c_{\alpha\beta,\alpha'\beta'} = g\delta_{\alpha\alpha'}\delta_{\beta\beta'}$. For simplicity we assume it to be spin independent, although in general its strength depends on the total spin of the two interacting atoms [13]. This approximation becomes exact in the case of $s = 1/2$ atoms, which we shall discuss below, since in this case only the spin-singlet channel contributes.

The dipole-dipole interaction $V^{dd}(\mathbf{q})$ is the Fourier transform of Eq. (1),

$$V^{dd}_{\alpha\beta,\alpha'\beta'}(\mathbf{q}) = 4\pi c_{dd}\Big(\frac{(\mathbf{q}\cdot\mathbf{s}_{\alpha\alpha'})(\mathbf{q}\cdot\mathbf{s}_{\beta\beta'})}{q^2} - \frac{\mathbf{s}_{\alpha\alpha'}\cdot\mathbf{s}_{\beta\beta'}}{3}\Big)\,, \qquad (3)$$

where $c_{dd} = d_0^2$ denotes the coupling constant of the dipole-dipole interaction and $\mathbf{s}_{\alpha\alpha'}$ is the matrix element of the spin operator between the basis spin functions $(s_m)_{\alpha\alpha'} = \sqrt{s(s+1)}C^{s\alpha}_{s\alpha'1m}$ with $s_{\pm1} = \mp(s_x \pm is_y)/\sqrt{2}$, $s_0 = s_z$, and the Clebsch-Gordan coefficient $C^{s\alpha}_{s\alpha'1m}$ in the notation of Ref. [14].

We assume for the moment that the ground state of the system is spin symmetric. In this case, the DDI does not contribute to the mean field, which implies that all spin components have the same spherical Fermi surface (note that, as shown in [4, 5], the Fermi surface deforms if the ground state is spin asymmetric, i.e., if the gas is fully or partially polarized). Then the occupation numbers are

given by $\rho_{\alpha\beta}(\mathbf{k}) = \langle c^\dagger_{\mathbf{k},\beta} c_{\mathbf{k},\alpha}\rangle = \delta_{\alpha\beta}\rho(\mathbf{k}) = \delta_{\alpha\beta}\theta(k_F - k)$, where $k_F$ denotes the Fermi momentum. The single-particle energies are $\varepsilon_{\mathbf{k}} = k^2/(2m) + 2sgn$, where $2sgn$ is the Hartree-Fock mean field, $n = k_F^3/(6\pi^2)$ being the density per spin state.

We will use RPA theory in order to investigate for which parameters the symmetric ground state gets unstable with respect to the formation of more interesting asymmetric phases. To that end, we calculate the spectrum of the collective zero-sound modes: A vanishing or imaginary frequency indicates an instability. Let us consider the retarded response function

$$i\Pi_{\alpha\beta,\alpha'\beta'}(\mathbf{k},\mathbf{k}',\mathbf{q},t-t') = \theta(t-t') \times \langle[c^\dagger_{\mathbf{k},\beta}(t)c_{\mathbf{k}+\mathbf{q},\alpha}(t), c^\dagger_{\mathbf{k}'+\mathbf{q},\alpha'}(t')c_{\mathbf{k}',\beta'}(t')]\rangle\,. \quad (4)$$

Within RPA, it is obtained as the solution of the integral equation

$$\Pi_{\alpha\beta,\alpha'\beta'}(\mathbf{k},\mathbf{k}',\mathbf{q},\omega) = \Pi^0(\mathbf{k},\mathbf{q},\omega)(2\pi)^3\delta(\mathbf{k}-\mathbf{k}')\delta_{\alpha\alpha'}\delta_{\beta\beta'} + \Pi^0(\mathbf{k},\mathbf{q},\omega)\sum_{\alpha_1\beta_1}\int\frac{d^3k_1}{(2\pi)^3}f_{\alpha\beta_1,\beta\alpha_1}(\mathbf{q},\mathbf{k}-\mathbf{k}_1) \times \Pi_{\alpha_1\beta_1,\alpha'\beta'}(\mathbf{k}_1,\mathbf{k}',\mathbf{q},\omega)\,, \quad (5)$$

where $f_{\alpha\beta,\alpha'\beta'}(\mathbf{q},\mathbf{k}-\mathbf{k}') = V_{\alpha\beta,\alpha'\beta'}(\mathbf{q}) - V_{\alpha\beta,\beta'\alpha'}(\mathbf{k}-\mathbf{k}')$ denotes the antisymmetrized matrix element of the interaction and $\Pi^0(\mathbf{k},\mathbf{q},\omega) = [\rho(\mathbf{k})-\rho(\mathbf{k}+\mathbf{q})]/(\omega-\varepsilon_{\mathbf{q}+\mathbf{k}}+\varepsilon_{\mathbf{k}}+i\eta)$ is the non-interacting response function.

In the limit $q \ll k_F$, the response function is concentrated at $k = k' = k_F$ and it depends only on the directions of $\mathbf{k}$ and $\mathbf{k}'$ and on the dimensionless quantity $\tilde{\omega} = m\omega/(k_F q)$ (without loss of generality we suppose that $\mathbf{q} = q\mathbf{e}_z$). As usual in Landau Fermi liquid theory, we expand the angular dependence in spherical harmonics $Y_{LM_L}$. A complication arises from the fact that the DDI is not diagonal in orbital angular momentum $L$ and spin $S$ of the excitation, but only in the total angular momentum $J$. We introduce a multi-index $\Lambda = LSJM$ and define the angular momentum projected response function as

$$\tilde{\Pi}_{\Lambda\Lambda'}(\tilde{\omega}) = \sum_{M_S M_L M_S' M_L'} C^{JM}_{LM_L SM_S} C^{J'M'}_{L'M_L' S'M_S'} \times \sum_{\alpha\beta\alpha'\beta'}(-1)^{s-\beta}C^{SM_S}_{s\alpha s-\beta}(-1)^{s-\beta'}C^{S'M_S'}_{s\alpha' s-\beta'} \times \int\frac{d^3k}{(2\pi)^3}\frac{d^3k'}{(2\pi)^3}Y^*_{LM_L}(\Omega_k)Y_{L'M_L'}(\Omega_k') \times \frac{(2\pi)^3}{mk_F}\Pi_{\alpha\beta,\alpha'\beta'}(\mathbf{k},\mathbf{k}',\mathbf{q},\omega)\,. \quad (6)$$

Then the RPA equation (5) reduces to a matrix equation:

$$\tilde{\Pi}_{\Lambda\Lambda'}(\tilde{\omega}) = \tilde{\Pi}^0_{\Lambda\Lambda'}(\tilde{\omega}) + \sum_{\Lambda_1\Lambda_2}\tilde{\Pi}^0_{\Lambda\Lambda_1}(\tilde{\omega})F_{\Lambda_1\Lambda_2}\tilde{\Pi}_{\Lambda_2\Lambda'}(\tilde{\omega})\,, \quad (7)$$

where

$$\tilde{\Pi}^0_{\Lambda\Lambda'}(\tilde{\omega}) = \delta_{SS'}\delta_{MM'}\sum_{M_L M_S}C^{JM}_{LM_L SM_S}C^{J'M}_{L'M_L SM_S} \times \int d\Omega Y^*_{LM_L}(\Omega)\frac{\cos\theta}{\tilde{\omega}-\cos\theta+i\eta}Y_{L'M_L}(\Omega)\,. \quad (8)$$

The Landau parameters $F_{\Lambda\Lambda'}$ can be decomposed into the direct ($D$) and exchange ($Ex$) contributions from the contact ($c$) and dipole-dipole ($dd$) interactions, $F_{\Lambda\Lambda'} = F^{c(D)}_{\Lambda\Lambda'} + F^{c(Ex)}_{\Lambda\Lambda'} + F^{dd(D)}_{\Lambda\Lambda'} + F^{dd(Ex)}_{\Lambda\Lambda'}$, and the corresponding explicit expressions read:

$$F^{c(D)}_{\Lambda\Lambda'} = \frac{mk_F g}{2\pi^2}(2s+1)\delta_{\Lambda\Lambda'}\delta_{L0}\delta_{S0}\delta_{J0}\delta_{M0}\,, \quad (9)$$

$$F^{c(Ex)}_{\Lambda\Lambda'} = -\frac{mk_F g}{2\pi^2}\delta_{\Lambda\Lambda'}\delta_{L0}\delta_{SJ}\,, \quad (10)$$

$$F^{dd(D)}_{\Lambda\Lambda'} = \frac{2mk_F c_{dd}}{\pi}s(s+1)(2s+1)\frac{2-3M^2}{9} \times \delta_{\Lambda\Lambda'}\delta_{L0}\delta_{S1}\delta_{J1}\,, \quad (11)$$

$$\begin{aligned} F^{dd(Ex)}_{\Lambda\Lambda'} = &-\frac{5mk_F c_{dd}}{\pi}s(s+1)(2s+1)\delta_{JJ'}\delta_{MM'} \\ &\times\sqrt{(2S+1)(2S'+1)(2L+1)(2L'+1)} \\ &\times(-1)^{S+J}\Bigg[(H_L+H_{L'})\begin{pmatrix}L & L' & 2\\ 0 & 0 & 0\end{pmatrix}\begin{pmatrix}1 & 1 & 2\\ 0 & 0 & 0\end{pmatrix} \\ &+2(-1)^L\sum_\ell(2\ell+1)H_\ell\begin{pmatrix}L & 1 & \ell\\ 0 & 0 & 0\end{pmatrix}\begin{pmatrix}L' & 1 & \ell\\ 0 & 0 & 0\end{pmatrix} \\ &\times\begin{Bmatrix}1 & 1 & 2\\ L & L' & \ell\end{Bmatrix}\Bigg]\begin{Bmatrix}L & S & J\\ S' & L' & 2\end{Bmatrix}\begin{Bmatrix}s & s & S\\ s & s & S'\\ 1 & 1 & 2\end{Bmatrix}\,, \end{aligned} \quad (12)$$

where the standard 3j, 6j and 9j symbols [14] have been used, and $H_0 = 0$, $H_n = \sum_{p=1}^n 1/p$ are the harmonic numbers.

At this point it seems necessary to add a comment about the direct term of the DDI in Eq. (11). As one can see from Eqs. (9)−(12), the contribution of the direct term of the DDI is the only one that depends on $M$. Since the Landau parameters are defined in the limit $\mathbf{q} \to 0$, one would expect that there should not be any preferred direction and therefore no dependence on $M$. In fact, for $\mathbf{q} = 0$, one sees immediately that $V^{dd}(\mathbf{q}=0) = \int d^3r V_{dd}(\mathbf{r}) = 0$, and therefore the direct term was omitted in the analysis of Ref. [15]. However, the DDI is discontinuous at $\mathbf{q} = 0$ because of its long-range nature, and as one can see from Eq. (3), $V^{dd}(\mathbf{q})$ depends only on the direction of $\mathbf{q}$ and therefore does not vanish in the limit $\mathbf{q} \to 0$. Since an instability occurs when the energy of a zero-sound mode (which exists only at small but non-zero $\mathbf{q}$) vanishes, we will include the direct term, as it was done in Ref. [16].

Equation (7) can now easily be solved by matrix inversion:

$$\tilde{\Pi}_{\Lambda\Lambda'}(\tilde{\omega}) = \sum_{\Lambda_1}(\mathcal{M}^{-1})_{\Lambda\Lambda_1}\tilde{\Pi}^0_{\Lambda_1\Lambda'}(\tilde{\omega})\,, \quad (13)$$

where

$$\mathcal{M}_{\Lambda\Lambda'}(\tilde{\omega}) = \delta_{\Lambda\Lambda'} - \sum_{\Lambda_1} \tilde{\Pi}^0_{\Lambda\Lambda_1}(\tilde{\omega}) F_{\Lambda_1\Lambda'}. \tag{14}$$

The onset of an instability is characterized by a vanishing excitation energy. We therefore consider the response function in the case $\tilde{\omega} = 0$. In this case, Eq. (8) reduces to $\tilde{\Pi}^0_{\Lambda\Lambda'}(0) = -\delta_{\Lambda\Lambda'}$, so that $\mathcal{M}_{\Lambda\Lambda'}(0) = \delta_{\Lambda\Lambda'} + F_{\Lambda\Lambda'}$. Stability requires that all eigenvalues of $\mathcal{M}(0)$ are positive [6, 16, 17]. Note that the Landau parameters $F_{\Lambda\Lambda'}$ and hence the stability matrix $\mathcal{M}$ are diagonal with respect to the total angular momentum $J$.

Let us now discuss the stability conditions for the special case of $s = 1/2$ atoms. In this case, the total spin $S$ of the excitations can be $S = 0$ or $S = 1$.

In the channel $J = 0$, there exist two uncoupled modes: $\Lambda = LSJM = 0000$ and $1100$. The $\Lambda = 0000$ mode is associated with the compressibility and an instability in this channel means that the system will collapse. The corresponding stability condition is independent of the DDI since $F^{\mathrm{dd(D)}}_{0000,0000} = F^{dd(Ex)}_{0000,0000} = 0$ and reads

$$mk_F g > -2\pi^2 \,. \tag{15}$$

For $\Lambda = 1100$, the only contribution comes from the exchange term of the DDI, since $F^{c(D)}_{1100,1100} = F^{\mathrm{c}(Ex)}_{1100,1100} = F^{dd(D)}_{1100,1100} = 0$. From Eq. (12) one obtains $F^{dd(Ex)}_{1100,1100} = mk_F c_{dd}/(2\pi)$, and since $c_{dd}$ is positive, this mode is always stable.

In the case of $J = 1$, there exist four modes: $\Lambda = 101M$, $111M$, $011M$ and $211M$, the last two being coupled to each other. For $\Lambda = 101M$, no instability can occur because $F_{1010,1010} = 0$.

The response in the $L = 0, S = 1$ (i.e., $\Lambda = 011M$) channel is related to the spin susceptibility [7, 8], and an instability in this channel therefore is an instability towards spontaneous magnetization. Since the $\Lambda = 011M$ and $211M$ channels are coupled, the stability condition is that the matrix

$$\mathcal{M}_{S=1,J=1,M}(0) = \begin{pmatrix} 1 + F_{011M,011M} & F_{011M,211M} \\ F_{011M,211M} & 1 + F_{211M,211M} \end{pmatrix} \tag{16}$$

has positive eigenvalues. This condition depends on both $g$ and $c_{dd}$. The relevant non-vanishing contributions from the exchange term of the DDI [Eq. (12)] are $F^{\mathrm{dd}(Ex)}_{011M,211M}/\sqrt{2} = F^{dd(Ex)}_{211M,211M} = mk_F c_{dd}/(12\pi)$. Note that the stability condition depends on $M$ because of the direct term of the DDI [Eq. (11)]. Physically, the $M = \pm 1$ and $M = 0$ modes correspond to transverse and longitudinal spin waves, respectively. For $M = \pm 1$, $F^{dd(D)}_{011M,011M}$ is negative, while for $M = 0$ it is positive. Therefore the $M = \pm 1$ modes become unstable before an instability appears in the $q = 0$ case without the direct term of the DDI.

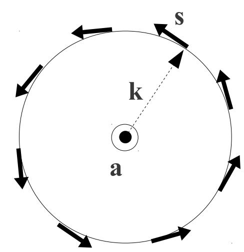


FIG. 1: Schematic representation of the spin texture induced by the perturbation (18) or created spontaneously if the system is unstable in the $\Lambda = 111M$ channel. The circle represents a section though the Fermi sphere. The vector $\mathbf{a}$ points towards the reader.

For $\Lambda = 111M$, the only non-vanishing Landau parameter is $F^{dd(Ex)}_{111M,111M}$ which is independent of $M$. The stability condition for this channel reads

$$mk_F c_{dd} < 4\pi. \tag{17}$$

Let us discuss the physical meaning of this instability. The mode $\Lambda = 111M$ can be excited by the following perturbation

$$H' = \sum_{\alpha\beta} \int \frac{d^3k}{(2\pi)^3} \mathbf{a} \cdot (\mathbf{s}_{\alpha\beta} \times \mathbf{k}) c^\dagger_{\mathbf{k}+\mathbf{q},\alpha} c_{\mathbf{k},\beta} \,, \tag{18}$$

where $\mathbf{a}$ characterizes the amplitude and orientation of the perturbation. This perturbation tends to turn the spin $\mathbf{s}$ of an atom into the direction perpendicular to the momentum $\mathbf{k}$ and to the vector $\mathbf{a}$. As a consequence, the Fermi surfaces of atoms whose spins point into the favored and unfavored directions split up and a spin texture in momentum space as shown in Fig. 1 is created. This is very similar to the so-called Rashba SOC [18, 19] which has recently been discussed also in the context of (two dimensional) systems of ultracold atoms [10]. If an instability occurs in this channel, this means that even in the absence of the perturbation (18), the system will spontaneously choose a direction $\mathbf{a}$ and create a spin texture with spins perpendicular to $\mathbf{a}$ and tangential to the Fermi surface, as shown in Fig. 1.

The different stability conditions are summarized in Fig. 2 which shows the stable regions as functions of the coupling constants $g$ and $c_{dd}$. The long-dashed vertical line indicates the boundary satisfying Eq. (15); in the region to the left of this line, the mean-field theory predicts a collapse. The short-dashed curve indicates the onset of the instability towards magnetization obtained from Eq. (16). Finally, the region above the horizontal solid line, obtained from Eq. (17), is unstable with respect to the spontaneous formation of SOC discussed above. Therefore, the stable region of the spin symmetric ground state is the gray area. The region where one can expect to find the spontaneous formation of SOC is

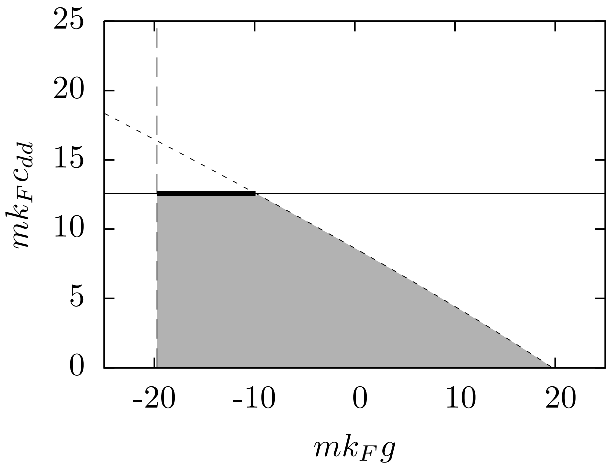


FIG. 2: The phase diagram for the stability of the spin symmetric ground state of the system as function of the coupling constants $g$ and $c_{dd}$. The lines indicate the onset of different instabilities: collapse (long-dashed vertical line), spontaneous magnetisation (short dashes), and spontaneous formation of a Rashba-like spin texture (solid horizontal line). The spin symmetric ground state is stable in the gray area.

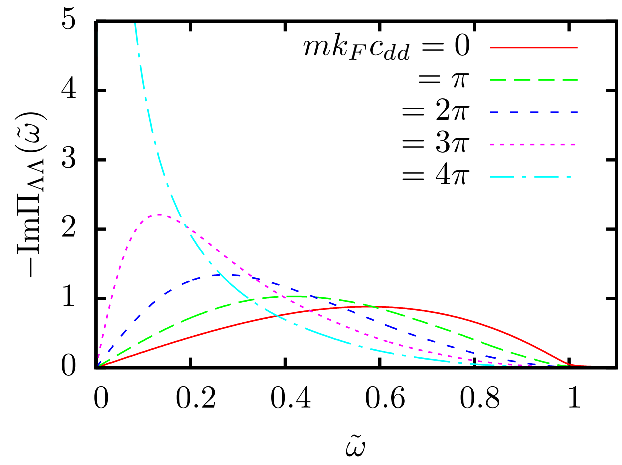


FIG. 3: The imaginary part of the response function (13) for $\Lambda = \Lambda' = 1110$ for various values of $c_{dd}$.

situated above the thick part of the solid line. For completeness, it should be mentioned that the modes with $J \geq 2$ do not change the phase diagram: For $J \geq 2$, the contact interaction does not contribute and the critical values for $c_{dd}$ are always larger than that given by Eq. (17).

Notice that the present stability analysis in the framework of RPA is only able to detect second-order phase transitions. To investigate the stability against first-order phase transition, one would have to use other methods, e.g., a variational ansatz as in Ref. [4].

So far we have only considered the response at $\tilde{\omega} = 0$. But using the RPA, we can also calculate the energy spectrum of the corresponding zero-sound modes. This is somewhat more difficult because at $\tilde{\omega} \neq 0$ the response function $\Pi^0_{\Lambda\Lambda'}$ is no longer diagonal with respect to $J$ (although it remains diagonal with respect to $M$) and, therefore, an infinite number of modes are coupled among one another in the matrix $\mathcal{M}(\tilde{\omega})$ of Eq. (14). To calculate numerically the response function, it is necessary to truncate the matrix at some value $J_{max}$. However, we found that convergence is practically reached at $J_{max} \sim 10$. As an example, we display in Fig. 3 the imaginary part of the response function in the channel $\Lambda = 1110$ for different values of the coupling constant $c_{dd}$. Since the Landau parameter $F_{1110,1110}$ is always negative, there is no zero-sound mode but only a broad particle-hole continuum. As one approaches the instability at $mk_Fc_{dd} = 4\pi$, the response gets more and more enhanced at low energy and at the instability it diverges at $\tilde{\omega} = 0$.

To summarize, we have applied mean-field and RPA theory to an unpolarized dipolar Fermi gas in the spin symmetric ground state. We have discussed the stability conditions in the special case of spin-1/2 atoms. In addition to the known collapse and spontaneous magnetization instabilities, we found that for certain values of $g$ and $c_{dd}$ the system gets unstable towards a phase with SOC where a Rashba-like spin texture around a spontaneously chosen axis in momentum space is formed.

An important subject for future studies will be to check whether the SOC phase survives also beyond the mean-field approximation. For instance, in reality the collapse predicted by the Hartree-Fock approach in the case of a strongly attractive contact interaction does never occur and the Fermi gas stays stable even in the unitary limit [20]. The stabilizing effect of short-range correlations was also discussed in the context of the instability towards spontaneous magnetization in the case of a strongly repulsive contact interaction (without DDI) [21]. In nuclear physics, short-range correlations due to the tensor force, which is similar to the DDI, are known to be important [22].

In addition, it is interesting to see what happens if the gas is trapped and not uniform. It will also be important to study not only the onset of the instability, but also the SOC phase itself and the competition between the different phases. Another interesting question is whether the SOC phase persists in (quasi-) two dimensional systems. In this case it would be possible to study, e.g., the spin Hall effect with cold atoms.

We thank D. Basko for interesting discussions and suggestions.


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